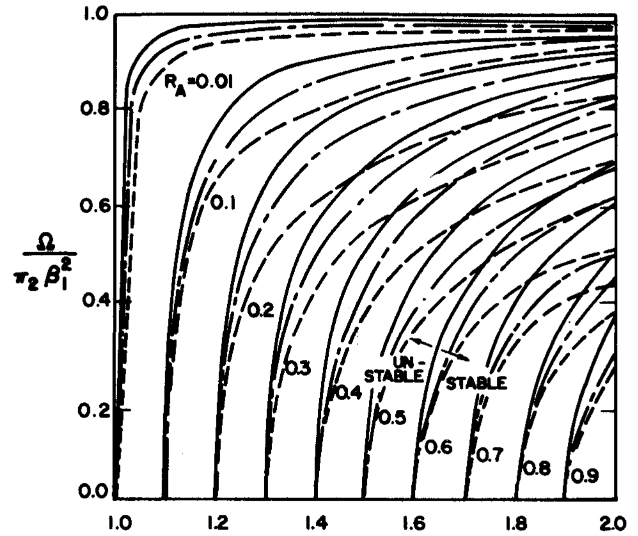

Fig. 2 Comparative study of stability regions, where ——— denotes results of present study, - - - - Meirovitch's results,[5] and ——-—— Meirovitch's results.[6]

$A_{\phi,i}$, etc., are the cofactors of the determinant of the Hessian matrix corresponding to the elements in the first row. The value of each determinant in Eqs. (13) and (14) can easily be calculated with the aid of a computer for any number of modal terms and the sums can be shown to be convergent. Note that as $n \to \infty$ the stability criteria given by Eqs. (11–14) provide an excellent means for studying the effects of modal series truncation on the accuracy of the stability region boundaries.

Discussion of Results and Conclusions

Computer results based on the stability conditions given by Eqs. (11–14) are presented in Fig. 2, where R_A is the ratio of the moment of inertia of the pair of antennas about the x axis to the moment of inertia of the rigid body about the same axis and $\Omega/\pi_2\beta_1{}^2$ is a dimensionless spin parameter. For R_A varying from 0.01 to 0.9 and $\Omega/\pi_2\beta_1{}^2$ varying from zero to 0.9, a single term in the modal series expansion yields results within 0.1% of those obtained with two or more modal terms. Thus, this analysis provides important evidence that the use of a single modal term satisfying both the geometric and natural boundary conditions of the problem yields surprisingly accurate stability region boundaries based on Liapunov's direct method even for rather flexible high-spin systems.

An identical mathematical model has been studied by Meirovitch[5] and the results are redrawn in Fig. 2 for purposes of comparison. Later, Meirovitch[6] developed a new and improved set of results for the same model which are also included in Fig. 2. It is easy to see that the present results contain larger stable regions than those of Refs. 5 and 6, with these differences magnifying as the spin rate and flexibility increase. These conclusions follow from the fact that Meirovitch's results are not developed directly from the Hamiltonian function, but are derived from a neighboring Liapunov testing function formed by adroitly employing the bounding properties of the Rayleigh quotient concept, thereby substantially reducing the algebraic complexities of the problem. The results presented herein serve as an important basis of comparison for assessing the accuracy of such approximate techniques, showing for this case that Meirovitch's approximation to the Hamiltonian gives conservative stability bounds. For a more detailed development of this research, see Ref. 11.

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Comment on the Equation for Aeroelastic Divergence of Unguided Launch Vehicles

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A MATRIX formulation of the problem of aeroelastic divergence of unguided launch vehicles using a discrete mass representation is presented in Ref. 1. The purpose of the present Note is to indicate an interesting feature of the equation of aeroelastic divergence. The final equation of aeroelastic divergence[1] is

$$\{F_n\} = q\overline{C_{N_\alpha}S}\left[\lceil 1 \rfloor + \left\{\frac{m_r}{M}\right\}\left\lfloor \frac{X_{cp}-X_r}{X_{cg}-X_{cp}} \right\rfloor + \left[\frac{C_{N_\alpha}S_r}{\overline{C_{N_\alpha}S}}\right]\{1\}\left\lfloor \frac{X_r-X_{cg}}{X_{cg}-X_{cp}} \right\rfloor\right]\left[\frac{C_{N_\alpha}S_r}{\overline{C_{N_\alpha}S}}\right][\rho_{r,n}]\{F_n\} \qquad (1a)$$

$$= q\overline{C_{N_\alpha}S}[A]\left[\frac{C_{N_\alpha}S_r}{\overline{C_{N_\alpha}S}}\right][\rho_{r,n}]\{F_n\} \qquad (1b)$$

where

$$[A] = \left[\lceil 1 \rfloor + \left\{\frac{m_r}{M}\right\}\left\lfloor \frac{X_{cp}-X_r}{X_{cg}-X_{cp}} \right\rfloor + \left[\frac{C_{N_\alpha}S_r}{\overline{C_{N_\alpha}S}}\right]\{1\}\left\lfloor \frac{X_r-X_{cg}}{X_{cg}-X_{cp}} \right\rfloor\right]$$

is transformation matrix (2)

[ ], ⌈ ⌋, { }, ⌊ ⌋ square, diagonal, column, and row matrices, respectively. $C_{N_\alpha}S_r$, F_r, m_r, and X_r are the product of normal-force-coefficient slope and panel area, total transverse force, mass, and distance from origin or reference station 0 of rth station, respectively.

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X_{cp} and X_{cg} are distances from origin 0 to the center of pressure, and the center of gravity, respectively. M is the total mass of structure. $\overline{C_{N_\alpha} S}$ is the product of total normal force coefficient slope and total reference area; q is dynamic pressure; $\rho_{r,n}$ is the total slope influence coefficient (slope at $X = X_r$ is due to a unit load at $X = X_n$ when cantilevered at $X = 0$); and p is the total number of discrete elements of discrete mass representation of the structure.

It is found that matrix $[A]$ of Eq. (2) is a numerically invariant quantity regardless of the location of the reference point 0. The proof of the preceding statement is as follows. Reference 1 selects the reference station at one of the discrete masses by virtue of which the matrix size will be reduced by one less than the number of discrete stations comprising the discrete mass system. Let us take a new reference station at a distance a from previous reference station 0. Then new X_r', the distance of rth station from the new reference station 0′ in terms of X_r is related by

$$X_r' = X_r - a$$

or, in general,

$$\{X_r'\} = \{X_r - a\}$$

It is seen that

$$X_{cg}' - X_{cp}' = X_{cg} - X_{cp} = \text{constant} \tag{3a}$$

$$X_{cp}' - X_r' = X_{cp} - X_r \tag{3b}$$

$$X_r' - X_{cg}' = X_r - X_{cg} \tag{3c}$$

$$\overline{C_{N_\alpha} S'} = \overline{C_{N_\alpha} S} = \sum_{r=1}^{p-1} C_{N_\alpha} S_r + C_{N_\alpha} S_o \tag{3d}$$

$$M' = M = \sum_{r=1}^{p-1} m_r + m_o \tag{3e}$$

Equations (3d) and (3e) are the sum of product of normal force-coefficient slope and panel areas of all stations; and total mass of structure, respectively, and are independent of the reference coordinate system.

The transformation matrix computed by selecting a new reference station at 0′ (let us denote it by $[A']$) is given as

$$[A'] = \left[\lceil 1 \rfloor + \left\{\frac{m_r}{M}\right\} \left\lfloor \frac{X_{cp}' - X_r'}{X_{cg}' - X_{cp}'} \right\rfloor + \left\lceil \frac{C_{N_\alpha} S_r}{\overline{C_{N_\alpha} S}} \right\rceil \{1\} \times \left\lfloor \frac{X_r' - X_{cg}'}{X_{cg}' - X_{cp}'} \right\rfloor \right]$$

Using Eq. (3)

$$[A'] = [A] \tag{4}$$

is the same as transformation matrix $[A]$ of Eq. (2), which is obtained by taking the reference station at 0. Thus, the transformation matrix $[A]$ is a numerically invariant quantity regardless of the location of the reference point 0.

This is because the transformation matrix $[A]$ is obtained by using the translational and rotational equilibrium conditions

$$\lfloor 1 \rfloor \{F_r\} + F_o = 0$$

and

$$\lfloor X_r \rfloor \{F_r\} = 0$$

respectively, and which does not depend on the coordinate system used for formulation.

Now with respect to the new reference station at 0′, Eq. (1) is written as

$$\{F_n'\} = q \overline{C_{N_\alpha} S} [A'] \left\lceil \frac{C_{N_\alpha} S_r}{\overline{C_{N_\alpha} S}} \right\rfloor [\rho_{r,n}'] \{F_n'\} \tag{5}$$

where $\rho_{r,n}'$ is the total slope influence coefficient with respect to the new reference station 0′.

Using Eq. (4) in Eq. (5)

$$\{F_n'\} = q \overline{C_{N_\alpha} S} [A] \left\lceil \frac{C_{N_\alpha} S_r}{\overline{C_{N_\alpha} S}} \right\rfloor [\rho_{r,n}'] \{F_n'\} \tag{6}$$

Equations (1) and (6) represent eigenvalue problem for a given structure using different coordinate systems, hence, will yield the same eigenvalues. The eigenvectors, in general, will represent the normalized mode shape with reference to the coordinate system used in the formulation. It is seen from Eqs. (1) and (6) that matrix $[A]$ computed at reference station 0 can be used together with the matrix of slope influence coefficients $[\rho_{r,n}]$, computed at either reference station 0 or 0′. Since $[A]$ is a numerically invariant quantity regardless of reference coordinate system, matrix $[A]$ computed at any point can be used in Eq. (1) together with any set of slope influence coefficients for the structure clamped at any other point. For the structure clamped at the Sth station $\rho_{r,n} = 0$ for $r \leq S \leq n$.

Proper choice of S will reduce considerably the amount of computations for the $\rho_{r,n}$'s. Note that matrix $[A]$ has to be calculated by suppressing the Sth row and column since the structure is clamped at the Sth discrete mass.

It is also seen that matrix $[A]$ can be very easily calculated choosing the reference station at center of gravity of the structure. Then the expression for $[A]$ simplifies to

$$[A] = \left[\lceil 1 \rfloor - \left\{\frac{m_r}{M}\right\} \lfloor 1 \rfloor + \left\{\frac{m_r}{M} - \frac{C_{N_\alpha} S_r}{\overline{C_{N_\alpha} S}}\right\} \left\lfloor \frac{X_r}{X_{cp}} \right\rfloor \right]$$

Note that then the X_r's are measured from the center of gravity as the reference station.

In the past, Refs. 2, 3 have shown the numerical invariance of similar transformation matrix in the case of vibration analysis of free-free structures. Reference 2 uses a lumped mass representation of the structure, whereas Ref. 3 uses a modified mass matrix method (then the mass matrix is full rather than a diagonal one as in the case of the lumped mass method). The way of formulation in all cases[1–3] is the same. Hence, in general, it is concluded here that if the formulation of the problem using matrix notations is done in the way given in Refs. 1–3, then the corresponding transformation matrix obtained for free-free structures would always be a numerically invariant quantity regardless of the location of the reference point.

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Experimental Study of Mixed Convection Heat Transfer in an MHD Channel

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Nomenclature

a = width of the channel
b = depth of the channel

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